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A LINEAR ACOUSTIC MODEL OF THE PASSIVE EFFECT OF THE TURBINE OF AN AUTOMOTIVE TURBOCHARGER

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Abstract

The turbine of an automotive turbocharger is essentially one acoustic element in the exhaust system which lies between the primary noise source, the gas pulsations through the exhaust valves, and the primary noise radiation element, the exhaust tailpipe orifice. As such, like every other acoustic element of the exhaust system, it has a passive effect on the propagation of the primary exhaust noise. Thus if a comprehensive model of the acoustic propagation through the entire exhaust system of a turbocharged engine is sought, an acoustic model of the turbine is a prerequisite.

This paper presents a preliminary attempt to create such a model. The model is a purely fluid mechanic one, without recourse to any empiricism such as a turbine map. The nonlinear equations of the fluid flow are developed and solved for steady flow, to determine the mean convective flow effects upon the noise. The full time-domain equations are then linearised and solved for a single frequency of sound.

Results are given from both the steady flow and the acoustic analyses. The latter are presented in terms of both transmission loss and four-pole parameters. The model is found to give a rational representation of the passive effect of a turbine rotor.

INTRODUCTION

To date, fluid mechanic models of turbochargers for automotive applications have concentrated upon the prediction of the effect of the turbocharger upon the overall performance of the engine. This is quite natural, after all this is the sole purpose for which a turbocharger is used. Many such models can be found in the literature, varying from simplistic steady-flow models to comprehensive pulse-flow models, see for example [1-5]. A common alternative to trying to predict the effect of the turbocharger is to use measured data in the form of turbo-compressor and turbine maps to account for the effect of the turbocharger within an overall engine performance model.

Although the purpose of the turbocharger is to increase the power output of a given engine, the presence of the turbine and compressor within the exhaust and intake system respectively affects the propagation of sound through these systems. Thus in order to predict the acoustic performance of the entire exhaust or intake system on a turbocharged engine, an acoustic model of the turbine or turbo-compressor is required. Now the pulse-flow models developed for engine performance prediction, as mentioned above, effectively include the acoustic performance, since the fluid pulsations are the sound waves. However it is conventional in the modelling of the acoustic performance of exhaust and intake systems to make use of linear, frequency-domain analysis, since this simplified form of analysis enables one to model complex silencer configurations and produces accurate results very efficiently.

This paper represents, to the best knowledge of the authors, the first attempt to produce a linear acoustic model of the turbine of an automotive turbocharger. The turbine can be considered as a noise source, especially at the high frequencies associated with blade-passing. However at the low frequencies of dominant engine exhaust noise, namely engine firing frequency and its first few harmonics, the active noise generation of the turbine is insignificant and thus attention is focused upon the passive response of the turbine to the acoustic waves generated by the in-cylinder explosions which propagate through the gas flow within the exhaust system. Since attention is restricted to low frequency sound waves of long wavelength, we are able to make a further simplification and to consider only one-dimensional wave propagation.

GOVERNING EQUATIONS

The turbine can be considered as being composed of three separate sections, a volute, a rotor and a diffuser, see Figure 1.

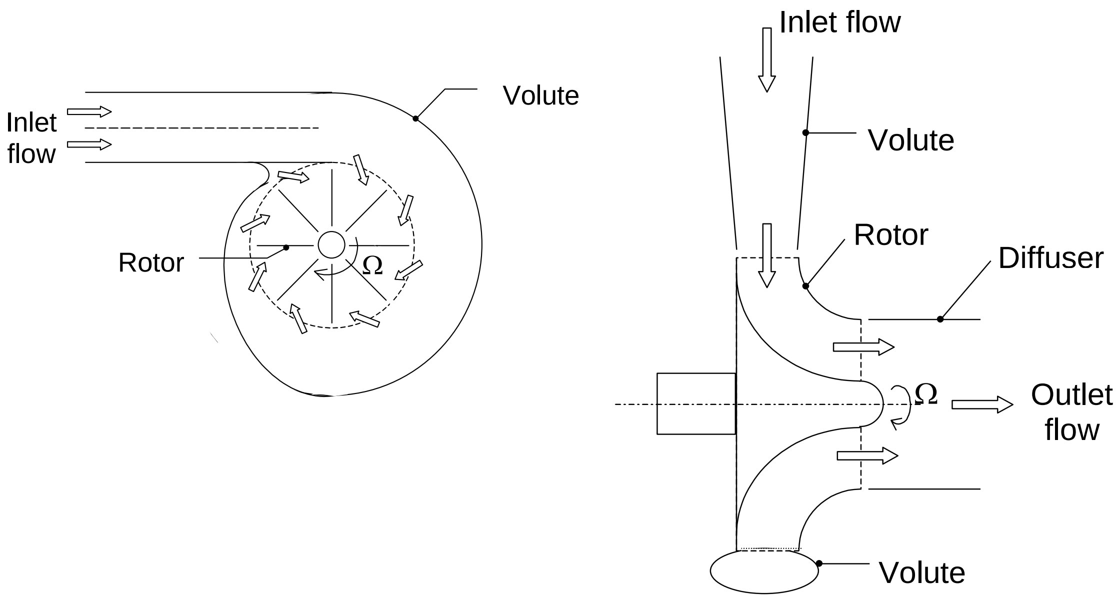

Figure 1. Sections of the Turbine

In each section, we require the equations for one-dimensional flow in direction x through a tube of varying cross-sectional area $A(x)$. The rotor consists of a number, say n, of identical curved blades with n flow passages between them. We consider the entire flow through the volute and diffuser to be distributed evenly between the n rotor passages and for the local flow direction to be that of the blades. If, within the rotor, we consider coordinate x to rotate with the rotor at angular velocity Ω and for the flow velocity V here to be the velocity relative to the rotor, then the full nonlinear time-dependent equations of conservation of mass, momentum and energy pertinent to all three sections of the turbine can be written as [4]:

$$\frac{\partial}{\partial t}\begin{bmatrix}\rho \\ m \\ e\end{bmatrix} + \frac{\partial}{\partial x}\begin{bmatrix} m \\ mV + p \\ hV\end{bmatrix} = \begin{bmatrix} -\dfrac{m}{A}\dfrac{\mathrm{d}A}{\mathrm{d}x} \\ -\dfrac{mV}{A}\dfrac{\mathrm{d}A}{\mathrm{d}x} + (B - F) \\ -\dfrac{hV}{A}\dfrac{\mathrm{d}A}{\mathrm{d}x} + mB \end{bmatrix} \tag{1}$$

where ρ is the density, p is the pressure, $m = \rho V$, $e = \dfrac{\rho V^2}{2} + \dfrac{p}{(\gamma - 1)}$, $h = e + p$, F is the wall friction per unit volume and $B = \rho \Omega^2 r \dfrac{\mathrm{d}r}{\mathrm{d}x}$. Here γ is the ratio of specific heats, $r(x)$ is the radial distance of the 1-D flow line from the axis of rotation, and of course $B = 0$ in the volute and diffuser. It is assumed that the exhaust gas is a perfect gas.

Steady-Flow Analysis

In the first instance, steady-flow solutions to equation (1) are sought in order to determine the background fluid properties for all x since these govern the propagation and convection of the sound waves. Equation (1), with the time derivative set to zero, becomes a set of three coupled first-order nonlinear ordinary differential equations. We assume that all geometrical data is known, in particular $A(x)$ throughout the turbine and $r(x)$ through the rotor. All the fluid properties at the inlet to the volute are assumed given. Solution for the flow properties as a function of distance x throughout each section of the turbine then follows from use of a two-step, predictor-corrector forward finite difference scheme. Matching conditions at the volute-rotor and rotor-diffuser boundaries are also required in order to progress the solution from one section of the turbine to the next. At both boundaries, continuity of pressure, temperature and mass flow rate is enforced. At the volute-rotor boundary, the outlet flow angle from the volute is assumed known and, together with the assumption that the flow through the rotor follows the blades, this is sufficient to determine the rotor speed. It is possible to determine the overall performance parameters from this analysis, such as the torque and power output of the rotor, but it must be stressed that this is not the purpose of the steady flow analysis. The only reason that a steady flow analysis is required for the present work is to determine the fluid properties of the steady flow with distance x through the turbine in order to subsequently calculate the propagation of sound waves through this medium.

Acoustic Analysis

Each fluid property of equations (1) is written as the sum of a steady-flow value and an acoustic value, e.g. $\rho = \bar{\rho}(x) + \rho'(x,t)$. We also assume that the acoustic disturbance is isentropic, i.e. $p' = \rho' \bar{c}^2$, which removes the requirement to consider the energy equation within (1) in this section. The steady flow equations are then subtracted from the overall equations and the equations are linearised on the basis that acoustic values are small enough for products of acoustic variables to be negligible in comparison to first-order terms. Once the equations are linearised it is sufficient to consider a single harmonic component of frequency ω say. Thus equations (1) reduce to:

$$\begin{bmatrix} \overline{\rho c} & \overline{M} \\ \overline{\rho c}\,\overline{M} & 1 \end{bmatrix} \begin{bmatrix} \dfrac{\mathrm{d}V'}{\mathrm{d}x} \\ \dfrac{\mathrm{d}p'}{\mathrm{d}x} \end{bmatrix} = \begin{bmatrix} \mathrm{i}kp' - p'\dfrac{\mathrm{d}\overline{M}}{\mathrm{d}x} - V'\overline{c}\dfrac{\mathrm{d}\overline{\rho}}{\mathrm{d}x} - \left(\overline{M}p' + \overline{\rho c}V'\right)\dfrac{1}{A}\dfrac{\mathrm{d}A}{\mathrm{d}x} \\ \mathrm{i}k\overline{\rho c}V' - \left(\overline{M}p' + \overline{\rho c}V'\right)\dfrac{\mathrm{d}\overline{M}}{\mathrm{d}x} + \left(\dfrac{\overline{\Omega}^2}{\overline{c}^2}p' + 2\overline{\rho}\,\overline{\Omega}\Omega'\right) r\dfrac{\mathrm{d}r}{\mathrm{d}x} \end{bmatrix} \tag{2}$$

where $\overline{M} = \overline{V}/\overline{c}$ is the Mach number and $k = \omega/\overline{c}$ is the wavenumber.

Exactly the same geometrical data for the turbine is needed here as for the steady flow analysis, and in addition the steady flow properties are now assumed known. Thus we are left with a pair of coupled first-order differential equations to solve for the variation of acoustic pressure and velocity throughout the turbine. The equations are in fact solved twice, for two different sets of inlet boundary conditions, namely $\{p' = 1, V' = 0\}$ and $\{p' = 0, V' = 1\}$. Once again, in each case, solution is effected by means of a two-step, predictor-corrector forward finite difference scheme, with matching conditions at the volute-rotor and rotor-diffuser boundaries in order to progress the solution from one section of the turbine to the next. At both boundaries, continuity of acoustic pressure and acoustic mass flow rate is enforced. At the volute-rotor boundary, with the outlet flow angle from the volute assumed known as before, the fluctuation in rotor speed Ω' follows from the assumption that the flow through the rotor follows the blades.

Thus, for a given set of inlet conditions, the acoustic properties are evaluated throughout the turbine, and in particular can be noted at the outlet section of the turbine. In particular, using the separate solutions from the two sets of inlet boundary conditions, the four-pole parameters *A*, *B*, *C*, *D* of the acoustic transfer matrix of the overall turbine can be evaluated, where

$$\begin{bmatrix} p'_{in} \\ V'_{in} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} p'_{out} \\ V'_{out} \end{bmatrix}. \tag{3}$$

It is, of course, simple to find the transfer matrix for any intermediate section(s) of the turbine. Once the transfer matrix is known, any further acoustic property, e.g. transmission loss, follows simply.

RESULTS

The results given are for the turbocharger of a 12 litre, 460 hp diesel engine. A complete nonlinear time-dependent model of the engine and turbocharger system has been conducted by AVL using AVL BOOST software. In this software, the effects of the turbo-compressor and turbine are deduced by reference to their respective measured maps rather than from direct modelling as here.

Steady Flow Results

The flow parameters at inlet to the volute as required in the current model were taken from the AVL BOOST output at three different engine speeds, see columns marked (In) in Table 1. The flow distribution through the turbine was then evaluated from the current model and the flow parameters at the outlet, in the diffuser, were then compared with the values from AVL BOOST. These values in Table 1 from AVL BOOST are evaluated average values from the time-dependent data. In contrast, the AVL BOOST figures for rotor speed and turbine power are only estimated average values from figures of the time-dependent variation.

It is seen that, relative to the overall change from the inlet values, the difference in outlet values of pressure and velocity between AVL BOOST and the current model is small. There is more discrepancy in the temperature values. Likewise the average rotor speeds are identical within the accuracy of estimation of the AVL BOOST values, but there is significant variation in the figures for power output.

Engine speed	1000 rpm			1250 rpm			1500 rpm		
	AVL	LU	(In)	AVL	LU	(In)	AVL	LU	(In)
Outlet pressure [bar]	1.124	1.056	1.739	1.125	1.095	2.025	1.127	1.070	2.467
Outlet temp. [bar]	927.3	890.3	928.6	875.8	848.7	906.1	866.1	842.6	931.4
Outlet velocity [m/s]	64.9	64.2	88.1	85.8	84.0	104.1	105.2	107.0	114.0
Rotor speed [rpm]	82000	81186		89500	89331		100400	100600	
Turbine power [kW]	35	33.7		55	61.4		75	101.7	

Acoustic Results

The steady flow results of flow distribution through the turbine at the three different engine speeds above were used as input data to the acoustic analysis in order to evaluate the four-pole parameters of the transfer matrix at different frequencies. These values were used to determine the transmission loss of the turbine at the different engine speeds, as shown in Figure 2.

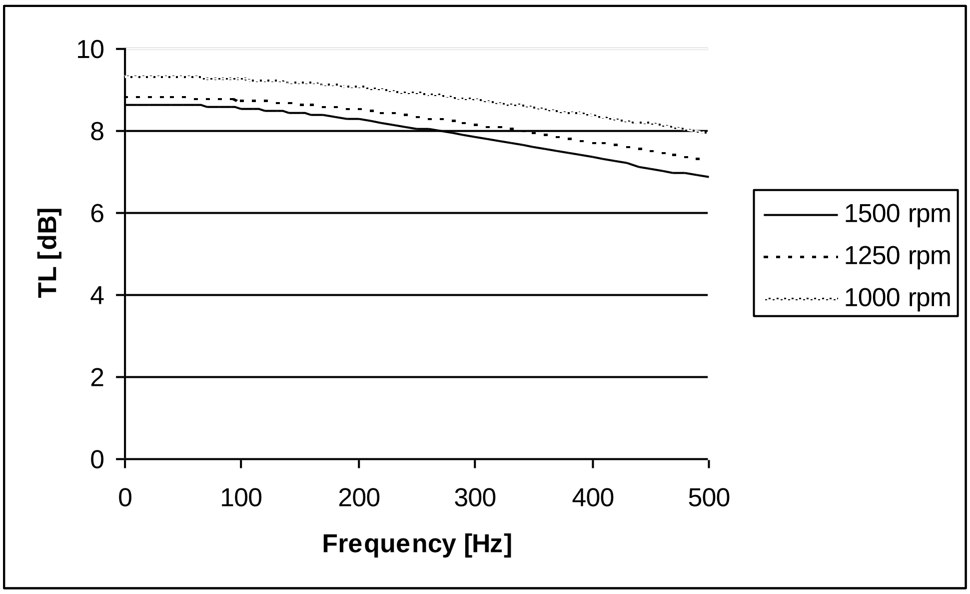

Figure 2. Transmission Loss of the Turbine

As yet we have no way to confirm the accuracy of these predictions, as experimental tests for verification are still in progress. However the nature of these results is similar to those of the transmission loss of an acoustically short conical pipe with similar changes of mean fluid properties, and hence the results are at least sensible.

CONCLUSIONS

A linear, passive acoustic model of the turbine of an automotive turbocharger has been developed that gives rational values of transmission loss. Part of the required input data for this model is the variation of mean fluid parameters through the turbine. These parameters were determined from a steady-flow fluid mechanic analysis of the turbine which gives reasonable results relative to a comprehensive engine model which incorporates the turbine and turbo-compressor maps. Since the basic equation set is the same for the steady-flow and the acoustic models, this gives some further justification for the results of the acoustic analysis.

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